For all questions, answer choice "E. NOTA" means none of the above answers is correct. Also, "DNE" means "does not exist".

1. Answer C. Converting the integrand to a power series yields the following: $\sum_{n=0}^{\infty}\frac{x^{2n}}{n!}$. Then, simply integrate the terms to get $\sum_{n=0}^{\infty}\frac{x^{2n+1}}{n!(2n+1)}$.

2. Answer E. None converge. ~~This limit of a summation looks suspiciously like a definite integral. We have the limit to infinity, we have the~~ $\cancel{\frac{b-a}{n}}$ ~~term (in this case~~ $\cancel{\frac{1}{n}}$~~), we simply need to identify the function and the limits. The function itself is being expressed in terms of series, in this case it is the series for~~ $\cancel{\frac{1}{1-x}}$ ~~which is~~ $\cancel{\sum_{k=0}^{\infty}x^k}$~~. Once identified, it is obvious that the limits are~~ $\cancel{-2}$ ~~and~~ $\cancel{-1}$~~, so all that is left is to evaluate the integral~~ $\cancel{\int_{-2}^{-1}\frac{1}{1-x}\,dx=\ln\left(\frac{1}{2}\right)-\ln\left(\frac{1}{3}\right)=\ln\left(\frac{3}{2}\right)}$

3. Answer C. A is the condition for divergence and B the condition for convergence.

4. Answer C. The interval in C ensures that the series meets the convergence conditions for the alternating series test, the easiest test to apply in this circumstance.

5. Answer D. The Ratio Test on the series $\sum_{n=1}^{\infty}\frac{2^n}{n!}$ shows it converges, therefore A converges absolutely. The nth term test for divergence is enough to show that B diverges.

6. Answer E. Function is not differentiable at x=0. $\cancel{\cos(x)=\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n}}{2n!}}$~~. Therefore, by simple substitution, the answer is B.~~

7. Answer E. Even a cursory look at the highest degree among the choices shows we are far from the mark. $\arctan(x)=\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+1}}{2n+1}$, so the 7th degree approximation for $f(x)$ above would be $4-\frac{4}{3}+\frac{4}{5}-\frac{4}{7}$ which is less than 3.

8. Answer B. $\ln(x)=\sum_{n=1}^{\infty}\frac{(-1)^{n+1}(x-1)^n}{n}$, but the given series is multiplied by -1, hence answer B.

9. Answer A. B diverges by the nth term test and C and D converge absolutely.

10. Answer C. While the Ratio Test and Root Test are both inconclusive (the respective limits for these test are both equal to 1), the Integral Test does show divergence.

11. Answer E. The power series for e^x is $\sum_{n=0}^{\infty}\frac{x^n}{n!}$, but the given series starts at n=1, therefore it is equal to $e-1$.

12. Answer A. Since this is the polynomial for $f'(x)$ the terms that concern us are the second and third terms. Recall that the third term of any Taylor polynomial would have a 2! On the denominator, so the real value of $f'''(1)$ would be -6. This minus the 5 from the second term gives us -11.

13. Answer C. This must be done differently than the preceding problem since the format is not as friendly. Recall that each derivative of a Taylor polynomial about $x = c$ matches the value of the same derivative of the original function at c. Therefore, $g'''(2) = P_4'''(2) = -78.5$ and $g''(2) = P_4''(2) = -87$. Therefore the answer is 8.5.

14. Answer A. In both cases the respective limits are greater than one, showing divergence.

15. Answer D. A and B diverge and are greater than the given series, making them unsuitable. C converges, but it less than the given series, also unsuitable. Only D is less than the given series and diverges, allowing us to determine that the given series diverges by Direct Comparison.

16. Answer D. For conditionally convergent series (which this one is) the sum has actually been shown, by the Riemann Series Theorem, to depend on the arrangement of the terms of the series.

17. Answer B. This is a well known sequence used to define the number e.

18. Answer B. This is a geometric series so the sum is given by $\frac{a_1}{1-r} = \frac{\frac{2}{15}}{1-\frac{2}{3}} = \frac{2}{5}$

19. Answer D. The summand can be rewritten using partial fraction decomposition as $\frac{1}{n} - \frac{1}{n+2}$. At this point it is obvious that this is a telescoping series with all the terms canceling except the first two positive terms: $1 + \frac{1}{2} = \frac{3}{2}$

20. Answer B. Let L be the limit, if it exist. It follows from the definition that $L = \sqrt{3+L}$. Solving for L gives us 2 and -1 as possible solutions, but the latter is extraneous.

21. Answer E. This process creates two geometric series. When one triangle is shaded, the first of these has an area of 8; the second has an area of 2; the third and area of $\frac{1}{2}$ and so on. The sum of these areas is $\frac{8}{1-\frac{1}{4}} = \frac{32}{3}$. When three triangles are shaded, the first set of these has an area of 12 (3 triangles each with an area of 4); the second set has an area of 3; the third and area of $\frac{3}{4}$ and so on. The sum of these areas is $\frac{12}{1-\frac{1}{4}} = 16$. Therefore the total area is $\frac{80}{3}$.

22. Answer C. Using the Integral Test shows that the integral converges as long as $-k+1<0$. So $k>1$.
23. Answer C. The remainder theorem for alternating series states that the maximum error is given by the first neglected term. If the tenth term is the first neglected term then the error is less than .01, therefore only 9 terms are needed.

24. Answer E. The power series for $\sin(x) = \sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+1}}{(2n+1)!}$. For $x=2$ the series becomes $\sum_{n=0}^{\infty}\frac{(-1)^n 2^{2n+1}}{(2n+1)!} = -\sum_{n=0}^{\infty}\frac{(-4)^{n+1}}{2(2n+1)!} = \sin(2)$. Therefore the sum of the given series is $2\sin(2)$.

25. Answer E. This is a case of the Limit Comparison test which states that under the given conditions either C or D is true, but it is impossible to determine which without more information.

26. Answer E. A converges conditionally (this can be seen by using the Limit Comparison Test to the harmonic series).

27. Answer C. A can be compared to $\sum_{n=1}^{\infty}\frac{\sqrt{n}}{n^2}$ and the Ratio Test can be used on B.

28. Answer C. A is a p-series with $p>1$ and B may be compared to a p-series.

29. Answer C. Both can be shown by the Root Test.

30. Answer B. A diverges by direct comparison to $\sum_{n=1}^{\infty}\frac{9^n}{7^n}$ and B converges by direct comparison to $\sum_{n=1}^{\infty}\frac{8}{n\sqrt{n}}$.